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Permanent Magnet Synchronous Motors are Globally Asymptotically Stabilizable with PI Current Control ⋆

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Abstract

This note shows that the industry standard desired equilibrium for permanent magnet synchronous motors (*i.e.*, maximum torque per Ampere) can be globally asymptotically stabilized with a PI control around the current errors, provided some viscous friction (possibly small) is present in the rotor dynamics and the proportional gain of the PI is suitably chosen. Instrumental to establish this surprising result is the proof that the map from voltages to currents of the incremental model of the motor satisfies some passivity properties. The analysis relies on basic Lyapunov theory making the result available to a wide audience.

Key words: Motor control, PI control, passivity theory, nonlinear control

1 Introduction

Control of electric motors is achieved in the vast majority of commercial drives via nested loop PI controllers [10, 11, 20]: the inner one wrapped around current errors and an external one that defines the desired values for these currents to generate a desired torque—for speed or position control. The rationale to justify this control configuration relies on the, often reasonable, assumption of time-scale separation between the electrical and the mechanical dynamics. In spite of its enormous success, to the best of our knowledge, a rigorous theoretical analysis of the stability of this scheme has not been reported. The main contribution of this paper is to (partially) fill-up this gap for the widely popular permanent magnet synchronous motors (PMSM), proving that the inner loop PI controller ensures global asymptotic stability (GAS) of the closed-loop, provided some viscous friction (possibly arbitrarily small) is present in the rotor dynamics, that the load torque is known and the proportional gain of the PI is suitably chosen, *i.e.*, sufficiently high. The assumption of known load torque is later relaxed proposing an adaptive scheme that, in the spirit of the aforementioned outer-loop PI, generates, via the addition of a simple integrator, an estimate for it—preserving GAS of the new scheme.

Several globally stable position and velocity controllers for PMSMs have been reported in the control literature—even in the sensorless context, *e.g.*, [2, 12, 23, 24] and references therein. However, these controllers have received an, at best, lukewarm reception within the electric drives community, which overwhelmingly prefers the aforementioned nested-loop PI configuration. Several versions of PI schemes based on fuzzy control, sliding modes or neural network control have been intensively studied in applications journals, see [9] for a recent review of this literature. To the best of our knowledge, a rigorous stability analysis of all these schemes is conspicuous by its absence.

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The importance of disposing of a complete theoretical analysis in engineering practice can hardly be overestimated. Indeed, it gives the user additional confidence in the design and provides useful guidelines in the difficult task of commissioning the controller. The interest of our contribution is further enhanced by the fact that the analysis relies on basic Lyapunov theory, using the natural (quadratic in the increments) Lyapunov function. Various attempts to establish such a result for PMSMs have been reported in the literature either relying on linear approximations of the motor dynamics or including additional terms that cancel some nonlinear terms, see [5,6] and references therein—a standing assumption being, similarly to us, the existence of viscous friction.

The remainder of this paper is organised as follows. The models of the PMSM are given in Section 2. The problem formulation is introduced in Section 3. The passivity of the PMSMs incremental model and the PI controller are established in Section 4. The main stability results are provided in Section 5. Some concluding remarks and discussion of future research are given in Section 6.

Notation. For $x \in \mathbb{R}^n$, $A \in \mathbb{R}^{n\times n}$, $A > 0$ we denote $|x|^2 = x^\top x$, $\|x\|_A^2 := x^\top Ax$. For the distinguished vector $x^\star \in \mathbb{R}^n$ and a mapping $\mathcal{C} : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, we define the constant matrix $\mathcal{C}^\star := \mathcal{C}(x^\star)$.

Caveat Emptor. Due to page limitation constraints this is an abridged version of the full paper, which may be found in [18].

2 Motor Models

In this section we present the motor model, define the desired equilibrium and give its incremental model.

2.1 Standard dq model

The dynamics of the surface-mounted PMSM in the dq frame is described by [10,21]:

$$\begin{aligned} L_d\frac{di_d}{dt} &= -R_s i_d + \omega L_q i_q + v_d \\ L_q\frac{di_q}{dt} &= -R_s i_q - \omega L_d i_d - \omega\Phi + v_q \\ J\frac{d\omega}{dt} &= -R_m\omega + n_p\left[(L_d - L_q)i_d i_q + \Phi i_q\right] - \tau_L \end{aligned} \tag{1}$$

where i_d, i_q are currents, v_d, v_q are voltage inputs, ω is the electrical angular velocity [1], $\frac{2n_p}{3}$ is the number of pole pairs, $L_d > 0, L_q > 0$ are the stator inductances, $\Phi > 0$ is the back emf constant, $R_s > 0$ is the stator resistance, $R_m > 0$ is the viscous friction coefficient, $J > 0$ is the moment of inertia and τ_L is a constant load torque.

Defining the state and control vectors as

$$x := \begin{bmatrix} i_d \\ i_q \\ \omega \end{bmatrix},\ u := \begin{bmatrix} v_d \\ v_q \end{bmatrix}$$

the system (1) can be written in compact form as

$$\mathcal{D}\dot{x} + [\mathcal{C}(x) + \mathcal{R}]x = Gu + d,$$

where

$$\mathcal{D} := \begin{bmatrix} L_d & 0 & 0 \\ 0 & L_q & 0 \\ 0 & 0 & \frac{J}{n_p} \end{bmatrix} > 0, \mathcal{R} := \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & \frac{R_m}{n_p} \end{bmatrix} > 0$$

$$\mathcal{C}(x) := \begin{bmatrix} 0 & 0 & -L_q x_2 \\ 0 & 0 & L_d x_1 + \Phi \\ L_q x_2 & -(L_d x_1 + \Phi) & 0 \end{bmatrix} = -\mathcal{C}^\top(x)$$

$$G := \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},\ d := \begin{bmatrix} 0 \\ 0 \\ -\frac{\tau_L}{n_p} \end{bmatrix},$$

Besides simplifying the notation, the interest of the representation above is that it reveals the power balance equation of the system. Indeed, the total energy of the motor is

$$H(x) = \frac{1}{2}x^\top\mathcal{D}x,$$

whose derivative yields

$$\underbrace{\dot{H}}_{stored\ power} = -\underbrace{x^\top\mathcal{R}x}_{dissipated} + \underbrace{y^\top u}_{supplied} - \underbrace{x_3\frac{\tau_L}{n_p}}_{extracted}, \tag{2}$$

where we used the skew-symmetry of $\mathcal{C}(x)$ and defined the currents as outputs, that is,

$$y := G^\top x = \begin{bmatrix} i_d \\ i_q \end{bmatrix}.$$

The current-feedback PI design is analysed in this paper viewing it as a passivity-based controller (PBC)—a term that was coined in [15]—where the main idea is to preserve a power balance equation like the one above

[1] Related with the rotor speed ω_m via $\omega = \frac{2n_p}{3}\omega_m$

but now with a new stored energy and a new dissipation term. This objective is accomplished in two steps, the shaping of the systems energy to give it a desired form, *i.e.*, to have a minimum at the desired equilibrium, and the injection of damping. The shaped energy function qualifies, then, as a Lyapunov function that ensures stability of the equilibrium, which can be rendered asymptotically stable via the damping injection.

Remark 1 See [16, 25] for additional discussion on the general theory of PBC and its practical applications and [1, 27] for some recent developments on PID-PBC.

2.2 Incremental model

The industry standard desired equilibrium is the maximum torque per Ampere value defined as

$$x^\star := \text{col}\left(0, \frac{1}{n_p\Phi}(\tau_L + R_m\omega^\star), \omega^\star\right), \tag{3}$$

where $\omega^\star$ is the desired electrical speed. With respect to this equilibrium we define the incremental model

$$\begin{aligned} \mathcal{D}\dot{\tilde{x}} + \mathcal{C}(x)\tilde{x} + [\mathcal{C}(x) - \mathcal{C}^\star]x^\star + \mathcal{R}\tilde{x} &= G\tilde{u} \\ \tilde{y} &= G^\top \tilde{x}, \end{aligned} \tag{4}$$

where $\tilde{(\cdot)} := (\cdot) - (\cdot)^\star$, $\mathcal{C}^\star := \mathcal{C}(x^\star)$, and we used the fact that

$$\begin{aligned} (\mathcal{C}^\star + \mathcal{R})x^\star &= Gu^\star + d \\ y^\star &= G^\top x^\star, \end{aligned}$$

with

$$u^\star = \begin{bmatrix} -\frac{1}{n_p\Phi}L_q\omega^\star(\tau_L + R_m\omega^\star) \\ \Phi\omega^\star + \frac{1}{n_p\Phi}R_s(\tau_L + R_m\omega^\star) \end{bmatrix}.$$

Note that

$$\tilde{y} = \begin{bmatrix} x_1 \\ x_2 - \frac{1}{n_p\Phi}(\tau_L + R_m\omega^\star) \end{bmatrix}. \tag{5}$$

3 Problem Formulation

We are interested in the paper in giving conditions for GAS of a PI controller wrapped around the currents i_d, i_q, which are assumed to be measurable. We consider two different scenarios.

S1 Known τ_L, Φ and R_m and "classical" PI

$$\begin{aligned} \dot{x}_c &= \tilde{y} \\ u &= -K_I x_c - K_P\tilde{y}, \end{aligned} \tag{6}$$

with $\tilde{y}$ defined in (5) and $K_I, K_P > 0$.

S2 Unknown τ_L but verifying the following (reasonable) assumption.

Assumption 1 A positive constant $\tau_L^{\max}$ such that

$$|\tau_L| \leq \tau_L^{\max},$$

is known.

Moreover, we assume that ω is measurable and, besides knowing the parameters Φ and R_m, it is also assumed that L_d, L_q and J are known. [2] In this scenario, we consider the adaptive PI controller

$$\begin{aligned} \dot{x}_c &= \begin{bmatrix} x_1 \\ x_2 - \hat{x}_2^\star \end{bmatrix} \\ u &= -K_I x_c - K_P \begin{bmatrix} x_1 \\ x_2 - \hat{x}_2^\star \end{bmatrix}, \end{aligned} \tag{7}$$

with $K_I, K_P > 0$, where $\hat{x}_2^\star$ is an estimate of the reference q-current $x_2^\star$, generated from an estimator of the simple integral form

$$\begin{aligned} \dot{\chi} &= f(x, \chi) \\ \hat{x}_2^\star &= h(x, \chi), \end{aligned} \tag{8}$$

with $\chi \in \mathbb{R}$, which is to be designed.

In both scenarios we want to prove that there exists a positive-definite gain matrix $K_P^{\min}$ such that the PMSM model (1) in closed-loop with the PI controller (6) or (7) with $K_P \geq K_P^{\min}$ has a GAS equilibrium at $(x^\star, x_c^\star, \chi^\star)$ for some $x_c^\star \in \mathbb{R}^2$ and $\chi^\star \in \mathbb{R}$ such that $h(x^\star, \chi^\star) = x_2^\star$. Moreover, in the second scenario, $K_P^{\min}$ should not depend on τ_L, but only on the bound given in Assumption 1.

Remark 2 As indicated in the introduction, in practice the reference value for x_2 is generated with an outer-loop PI around speed errors, that is,

$$\begin{aligned} \dot{\chi} &= \tilde{x}_3 \\ \hat{x}_2^\star &= -a_I\chi - a_P\tilde{x}_3, \end{aligned} \tag{9}$$

with $a_I, a_P > 0$. Unfortunately, the stability analysis of this configuration is far from obvious and we will need to propose another form for the functions $f(x, \chi)$ and $h(x, \chi)$ in (8).

Remark 3 For the sake of completeness we also propose an estimator for the viscous friction coefficient R_m, which generates a consistent estimate under an excitation assumption. See Subsection 5.3.

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[2] As shown in Proposition 2, these additional assumptions are needed to design the estimator of τ_L.

4 Passivity Analysis

4.1 Dissipativity of the incremental model

In this section we give conditions under which the incremental model (4) satisfies a dissipation inequality of the form

$$\dot{U} \leq \epsilon|\tilde{y}|^2 + \tilde{y}^\top \tilde{u}. \tag{10}$$

with

$$U(\tilde{x}) = \frac{1}{2}\|\tilde{x}\|_{\mathcal{D}}^2. \tag{11}$$

for some $\epsilon \in \mathbb{R}$. If ϵ is negative it is then said that the incremental model of the system (1) is output strictly passive, if it is positive, then it is called output feedback passive, indicating the shortage of passivity [8,14,25]. [3]

Comparing (10) with the open-loop power balance equation (2) we see that, besides removing the term of extracted power, we have shaped the energy—assigning a minimum at the desired equilibrium $x^\star$—and replaced the damping term $x^\top \mathcal{R} x$ by $\epsilon|\tilde{y}|^2$. Notice that, if ϵ is positive, it is easy to add damping selecting a control $\tilde{u} = -K_P\tilde{y}$, with $K_P = k_p I_2 > 0$. Indeed, this yields a damping term $-(k_p-\epsilon)|\tilde{y}|^2$, with $k_p > \epsilon$ we ensure $\dot{U} \leq 0$—whence, stability of the equilibrium. As explained in Remark 6, a more clever option is to add an integral action, yielding a PI.

Lemma 1 Define the matrix

$$\mathcal{B} := \begin{bmatrix} 2R_s + 2\epsilon & (L_d - L_q)x_3^\star & -L_d x_2^\star \\ (L_d - L_q)x_3^\star & 2R_s + 2\epsilon & 0 \\ -L_d x_2^\star & 0 & 2\frac{R_m}{n_p} \end{bmatrix},$$

for some $\epsilon \in \mathbb{R}$. If $\mathcal{B} \geq 0$ the dissipation inequality (10) holds.

PROOF. Computing the derivative of (11) along the solutions of (4) we get

$$\begin{aligned} \dot{U} &= -\tilde{x}^\top[\mathcal{C}(x) - \mathcal{C}^\star]x^\star - \tilde{x}^\top \mathcal{R}\tilde{x} + \tilde{y}^\top\tilde{u} \\ &= -\frac{1}{2}\tilde{x}^\top(\mathcal{B} - 2\epsilon G G^\top)\tilde{x} + \tilde{y}^\top\tilde{u} \\ &= -\frac{1}{2}\tilde{x}^\top\mathcal{B}\tilde{x} + \epsilon|\tilde{y}|^2 + \tilde{y}^\top\tilde{u}, \end{aligned}$$

where we have used the fact that

$$[\mathcal{C}(x) - \mathcal{C}^\star]x^\star = \begin{bmatrix} 0 & -L_q x_3^\star & 0 \\ L_d x_3^\star & 0 & 0 \\ -L_d x_2^\star & 0 & 0 \end{bmatrix}\tilde{x},$$

to get the second identity and use the definition of $\tilde{y}$ given in (4) in the third identity. The proof is completed imposing the condition $\mathcal{B} \geq 0$.

Remark 4 Lemma 1 follows as a direct application of Proposition 1 and Remark 3 of [14], where passivity of the incremental model of general port-Hamiltonian systems with strictly convex energy function is studied. To make the present paper self-contained we have included a proof of the lemma.

Remark 5 A dissipativity analysis similar to Lemma 1 has been carried out within the context of transient stability of power systems in [17], for synchronous generators connected to a constant voltage source in [26] and [3]. In all these papers the shifted Hamiltonian of [8], which in these cases boils down to the natural incremental energy function, is also used to establish stability conditions—that involve the analysis of positivity of a damping matrix similar to $\mathcal{B}$.

4.2 Strict passivity of the PI controller

In this subsection we prove the input strict passivity of the PI controller. Although this result is very well-known [25,27], a proof is given here for the sake of completeness.

Lemma 2 Given any constant $y_c^\star \in \mathbb{R}^2$, define the error signal $\tilde{y}_c := y_c - y_c^\star$. The PI controller

$$\begin{aligned} \dot{x}_c &= u_c \\ y_c &= K_I x_c + K_P u_c, \end{aligned}$$

defines an input strictly passive map $u_c \mapsto \tilde{y}_c$, with storage function

$$H_c(\tilde{x}_c) := \frac{1}{2}\|\tilde{x}_c\|_{K_I}^2, \tag{12}$$

where $x_c^\star := -K_I^{-1}y_c^\star$. More precisely

$$\dot{H}_c = -\|u_c\|_{K_P}^2 + u_c^\top \tilde{y}_c.$$

PROOF. First, notice that, using the definition of $x_c^\star$ in y_c we have that

$$\tilde{y}_c = K_I\tilde{x}_c + K_P u_c. \tag{13}$$

Computing the derivative of H_c along the trajectories of (6) yields

$$\dot{H}_c = \tilde{x}_c^\top K_I u_c = u_c^\top(\tilde{y}_c - K_P u_c),$$

where we have used (13) in the second identity, which completes the proof.

[3] In [14, 25] the property of passivity of the incremental model is called shifted passivity.

Remark 6 The PI controller described above will be coupled with the PMSM via the (power-preserving) interconnection $u_c = \tilde{y}$ and $y_c = -u$. Lemma 2 shows the interest of adding an integral action: there is no need to know $u^\star$ to implement the controller.

5 Main Results

5.1 Stability of the standard PI controller

Proposition 1 Consider the PMSM model (1) in closed-loop with the PI controller (6), the integral gain $K_I > 0$ and the proportional gain[4] $K_P = k_p I_2 > 0$. There exists a positive constant $k_p^{\mathtt{min}}$ such that

$$k_p \geq k_p^{\mathtt{min}} \tag{14}$$

ensures that $(x^\star, x_c^\star)$ is a GAS equilibrium of the closed-loop system. For non-salient PMSM, *i.e.*, when $L_d = L_q$, the constant $k_p^{\mathtt{min}}$ can be chosen such that

$$k_p^{\mathtt{min}} > \frac{L_d^2}{4R_m n_p \Phi^2} \left(\tau_L + R_m|\omega^\star|\right)^2 - R_s \tag{15}$$

PROOF. Summing up (11) and (12) define the positive definite, radially unbounded, Lyapunov function candidate

$$W(\tilde{x}, \tilde{x}_c) := U(\tilde{x}) + H_c(\tilde{x}_c). \tag{16}$$

Computing its derivative we get

$$\dot{W} = -\frac{1}{2}\|\tilde{x}\|^2_{\mathcal{B}} + (\epsilon - k_p)|\tilde{y}|^2 = -\frac{1}{2}\|\tilde{x}\|^2_{\mathcal{R}_d}, \tag{17}$$

where we defined the matrix

$$\mathcal{R}_d := \begin{bmatrix} 2R_s + 2k_p & (L_d - L_q)\omega^\star & -L_d x_2^\star \\ (L_d - L_q)\omega^\star & 2R_s + 2k_p & 0 \\ -L_d x_2^\star & 0 & 2\frac{R_m}{n_p} \end{bmatrix}$$

From (17) we immediately conclude that if $\mathcal{R}_d > 0$, then the equilibrium $(x^\star, x_c^\star)$ is globally stable. Moreover, invoking Krasovskii's Theorem, we prove that the equilibrium is GAS because

$$\tilde{x}(t) \equiv 0 \Rightarrow \tilde{x}_c(t) \equiv 0.$$

The gist of the proof is then to prove the existence of the lower bound $k_p^{\mathtt{min}}$ that ensures positivity of $\mathcal{R}_d$.

Towards this end, we recall the following well-known (Schur complement) equivalence:

$$\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix} > 0 \quad \Leftrightarrow \quad C > 0 \text{ and } A - BC^{-1}B^\top > 0.$$

Directly applying this to $\mathcal{R}_d$ with

$$A := \begin{bmatrix} 2R_s + 2k_p & (L_d - L_q)\omega^\star \\ (L_d - L_q)\omega^\star & 2R_s + 2k_p \end{bmatrix},\ B := \begin{bmatrix} -L_d x_2^\star \\ 0 \end{bmatrix},$$

and $C := 2\frac{R_m}{n_p}$, shows that $\mathcal{R}_d > 0$ if and only if

$$(R_s + k_p)I_2 > \frac{1}{2}\begin{bmatrix} \frac{n_p L_d^2 |x_2^\star|^2}{2R_m} & (L_q - L_d)\omega^\star \\ (L_q - L_d)\omega^\star & 0 \end{bmatrix}. \tag{18}$$

This proves the existence of $k_p^{\mathtt{min}}$ such that, if (14) holds then $\mathcal{R}_d > 0$. In case $L_d = L_q$, $k_p^{\mathtt{min}}$ can be chosen as in (15).

5.2 An asymptotically stable adaptive PI controller

In applications the load torque τ_L, and consequently $x_2^\star$ are unknown. It is, therefore, necessary to replace its value above by an estimate, a task, that is accomplished in the proposition below.

Proposition 2 Consider the PMSM model (1) verifying Assumption 1 in closed-loop with the adaptive PI controller (7) with the estimator

$$\begin{aligned} J\dot{\chi} &= -R_m\omega + n_p\left[(L_d - L_q)i_d i_q + \Phi i_q\right] - \ell(\chi - \omega) \\ \hat{\tau}_L &= \ell(\chi - \omega) \\ \hat{x}_2^\star &= \frac{1}{n_p\Phi}(\hat{\tau}_L + R_m\omega^\star) \end{aligned} \tag{19}$$

where $\ell > 0$. Fix the proportional gain as $K_P = k_p I_2 > 0$.

There exists a positive constant $k_p^{\mathtt{min}}$—dependent only on $\tau_L^{\mathtt{max}}$—such that (14) ensures that $(x^\star, x_c^\star, \chi^\star)$, with $\chi^\star := \frac{\tau_L}{\ell} + \omega^\star$ is a GAS equilibrium of the closed-loop system.

PROOF. Similarly to the proof of Proposition 1, we first need to prove that $\mathcal{R}_d > 0$. This follows immediately invoking (18) and noting that, from the definition of the equilibria in (3), we have

$$\frac{1}{n_P\Phi}(|\tau_L| + R_m|\omega^\star|) \geq |x_2^\star|.$$

[4] K_P is taken of this particular form to simplify the presentation of the main result—this choice is done without loss of generality.

Thus, a $k_p^{\mathtt{min}}$ that depends only on $\tau_L^{\mathtt{max}}$, can readily be defined.

We proceed now to prove that the estimator (19) generates an exponentially convergent estimate of τ_L. Defining the estimation error $e_{\tau_L} := \hat{\tau}_L - \tau_L$, the error dynamics yields

$$\dot{e}_{\tau_L} = -\frac{\ell}{J} e_{\tau_L}, \tag{20}$$

which is clearly exponentially stable for all $\ell > 0$.

To simplify the presentation of the analysis of the overall error dynamics let us define the reference output error signal

$$e_{y^\star} := \hat{y}^\star - y^\star = \begin{bmatrix} 0 \\ \hat{x}_2^\star - x_2^\star \end{bmatrix} = \frac{1}{n_P \Phi} \begin{bmatrix} 0 \\ e_{\tau_L} \end{bmatrix},$$

which replaced in (7) yields

$$\begin{aligned} \dot{\tilde{x}}_c &= \tilde{y} - e_{y^\star} \\ \tilde{u} &= -K_I \tilde{x}_c - K_P(\tilde{y} - e_{y^\star}) \end{aligned}$$

The closed-loop is then a cascaded dynamics of the form

$$\begin{aligned} \dot{e}_{y^*} &= -\frac{\ell}{n_P \Phi J} e_{y^*} \\ \dot{\xi} &= f(\xi) + \begin{bmatrix} \mathcal{D}^{-1} G K_P \\ -I_2 \end{bmatrix} e_{y^\star} \end{aligned} \tag{21}$$

with $\xi := \mathrm{col}(\tilde{x}, \tilde{x}_c)$ and the dynamics $\dot{\xi} = f(\xi)$ has the origin as a GAS equilibrium.

The GAS proof is completed invoking Theorem 1 of [19] that shows that the cascaded system is globally stable, which implies that all trajectories are bounded. GAS follows immediately from the well-known fact [22] that the cascade of two GAS systems is GAS if all trajectories are bounded. [5]

5.3 *A globally convergent estimator of R_m*

In the lemma below we show that it is possible to add an adaptation term to estimate the friction coefficient R_m, that is usually uncertain, provided some excitation conditions are satisfied.

Lemma 3 Consider the mechanical equation in (1) and the gradient estimator

$$\dot{\hat{R}}_m = \gamma \phi (z - \hat{R}_m \phi), \tag{22}$$

with $\gamma > 0$ an adaptation gain and the measurable signals

$$\begin{aligned} z &:= \frac{\beta p^2}{(p+\alpha)^2}[J\omega] + \frac{\beta p}{(p+\alpha)^2}[n_p(L_q - L_d) i_d i_q - n_p \Phi i_q] \\ \phi &:= \frac{\beta p}{(p+\alpha)^2}[\omega], \end{aligned} \tag{23}$$

where $p := \frac{d}{dt}$ and $\alpha, \beta > 0$. The following equivalence holds true

$$\phi \notin \mathcal{L}_2 \;\Leftrightarrow\; \lim_{t\to\infty} |\hat{R}_m(t) - R_m| = 0,$$

with $\mathcal{L}_2$ the space of square integrable functions.

PROOF. Applying the filter $\frac{\beta p}{(p+\alpha)^2}$ to the mechanical equation in (1), recalling that τ_L is constant, and using the definitions (23) yields the linear regression model

$$z = R_m \phi + \epsilon_t$$

where ϵ_t is an exponentially decaying term stemming form the filters initial conditions, which can be neglected without loss of generality. Replacing the equation above in (22) yields the error equation

$$\dot{e}_{R_m} = -\gamma \phi^2 e_{R_m}, \tag{24}$$

where $e_{R_m}; = \hat{R}_m - R_m$ is the parameter estimation error. The proof is completed integrating (24).

Remark 7 As always in estimation problems some kind of excitation on the signals must be imposed to guarantee convergence. In our case it is the condition of non-square integrability of ω, which is weaker than the more classical persistence of excitation assumption—in which case the convergence of the parameter error is exponential.

Remark 8 An alternative to the estimators presented above is to add a nonlinear integral action to compensate for both unknowns τ_L and R_m as done in [4]. In any case, both options considerably complicate the control law, a scenario that is beyond the scope of this paper. Also, although it is possible to carry out the stability analysis of the combination of the estimators of τ_L and R_m, we avoid this discussion for the aforementioned reason.

6 Conclusions and Future Research

We have established the practically interesting—though not surprising—result that the PMSM can be globally regulated around a desired equilibrium point with a simple (adaptive) PI control around the current errors, provided some viscous friction is present in the rotor dynamics and the proportional gain of the PI is suitably chosen.

[5] The first author expresses his gratitude to Antoine Chaillet, Denis Efimov and Elena Panteley for several discussions on the topic of cascaded systems.

The key ingredient to establish this result is the proof in Lemma 1 that the incremental model of the PMSM satisfies the dissipation inequality (10). Our main results are established with simple calculations and invoking elementary Lyapunov theory with the natural—quadratic in the increments—Lyapunov functions.

Some topics of current research are the following.

- From the theoretical viewpoint the main drawback of the results reported in the paper are the requirement of existence, and knowledge, of the friction coefficient R_m. As shown in Lemma 3 the requirement of knowing R_m can be relaxed—at the price of complicating the controller and requiring some excitation conditions. However the assumption of $R_m > 0$ seems unavoidable if we want to preserve a a simple PI structure, see Remark 8. It should be underscored, however, that from the practical viewpoint, the assumption that the mechanical dynamics has some static friction—that may be arbitrarily small—is far from being unreasonable.

- As discussed in [28] in the context of power systems, the absence of the outer-loop PI significantly deteriorates the transient performance of the inner-loop PI. A similar situation appears here for the PMSM. [6] . Unfortunately, the analysis of the classical outer-loop PI in speed errors (9) is hampered by the lack of a convergence proof of the estimation error.

- In the case of $L_d \neq L_q$ torque can be made even larger by an additional reluctance component $x_1^\star \neq 0$. The implications of this choice on the passivity of the incremental model remains to be investigated.

- The extension of the result to the case of salient PMSM is also very challenging—see [7] for the corresponding $\alpha\beta$ model.

- The lower bound on the proportional gain can be computed invoking the physically reasonable Assumption 1. However, the reference value for i_q is dependent on τ_L. As shown in Proposition 2 this problem can be solved using an adaptive PI, at the high cost of knowledge of the PMSM model parameters.

- Experimental results of PI current control abound in the literature and experiments of an observer, similar to (19), may be found in [21]. However, it would be interesting to validate experimentally the performance of the proposed adaptive PI and, in particular, investigate how it compares with the classical outer-loop speed PI (9).

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References

[1] S. Aranovskiy, R. Ortega and R. Cisneros, Robust PI passivity–based control of nonlinear systems and its application to port–Hamiltonian systems and temperature regulation, *International Journal of Robust and Nonlinear Control*, vol. 26, no. 10, pp. 2216-2231, July 2016.

[2] M. Bodson, J. Chiasson, R. Novotnak, and R. Ftekowski, High-performance nonlinear feedback control of a permanent magnet stepper motor, *IEEE Trans. Contr. Syst. Technol.*, vol. 1, no. 1, pp. 5-14, 1993.

[3] S. Y. Caliskan and P. Tabuada, Compositional transient stability analysis of multimachine power networks, *IEEE Transactions on Control of Network Systems*, vol. 1, no. 1, pp. 4-14, March 2014.

[4] J. Ferguson, A. Donaire and R. H. Middleton, Integral control of port-Hamiltonian systems: non-passive outputs without coordinate transformation, *IEEE Transactions on Automatic Control*, preprint, 2017. (`ArXiv 1703.07934`).

[5] V.M. Hernandez-Guzman and R.V. Carrillo-Serrano, Global PID position control of PM stepper motors and PM synchronous motors, *International Journal of Control*, vol. 84, no. 11, pp. 1807-1816, November 2011.

[6] V.M. Hernandez-Guzman and R. Silva, PI control plus electric current loops for PM synchronous motors, *IEEE Trans. Control Systems Technology*, vol. 19, no. 4, pp. 868-873, July 2011.

[7] S. Ichikawa, M. Tomita, S. Doki, and S. Okuma, Sensorless control of PMSM using on-line parameter identification based on systems identification theory, *IEEE Trans Ind. Electron.*, vol. 53, no. 2, pp. 363-373, April 2006.

[8] B. Jayawardhana, R. Ortega and E. García-Canseco, Passivity of nonlinear incremental systems: Application to PI stabilization of nonlinear RLC circuits, *Systems & Control Letters*, vol. 56, no. 9-10, pp. 618–622, 2007.

[9] J. Jung, V. Leu, T. Do, E. Kim and H. Choi, Adaptive PID speed control design for permanent magnet synchronous motor drives, *IEEE Trans Power Electronics*, vol. 30, no. 2, pp. 900-908, 2015.

[10] P. C. Krause, *Analysis of Electric Machinery*, McGraw Hill, New York, 1986.

[11] W. Leonhardt, *Control of Electrical Drives*, 2nd Edition, Springer, NY, 1996.

[12] J. Lee, J. Hong, K. Nam, R. Ortega, A. Astolfi and L. Praly, Sensorless control of surface-mount permanent magnet synchronous motors based on a nonlinear observer, *IEEE Transactions on Power Electronics*, vol. 25, no. 2, pp. 290-297, 2010.

[13] K. Liu and Z.Q. Zhu, Mechanical parameter estimation of PMSMs with aiding from estimation of rotor PM flux linkage, *IEEE Trans. Automatic Control*, vol. 51, no. 4, pp. 3115-3125, July-Aug. 2015.

[14] N. Monshizadeh, P. Monshizadeh, R. Ortega and A. van der Schaft, Conditions on shifted passivity of port-Hamiltonian systems, *Systems and Control Letters*, 2017, (submitted). (`arXiv:1711.09065`.)

[15] R. Ortega and M. Spong, Adaptive motion control of rigid robots: A tutorial, *Automatica*, vol. 25, no. 6, pp. 877-888, 1989.

[16] R. Ortega, A. Loria, P. J. Nicklasson and H. Sira-Ramirez, *Passivity-Based Control of Euler-Lagrange Systems*, Springer-Verlag, Berlin, Communications and Control Engineering, 1998.

[17] R. Ortega, A. Stanković and P. Stefanov, A passivation approach to power systems stabilization, *IFAC Symp Nonlinear Control Systems Design*, Enschede, Holland, July 1-3, 1998.

[18] R. Ortega, N. Monshizadeh, P. Monshizadeh, D. Bazylev and A. Pyrkin, PMSMs are globally asymptotically stabilizable with PI current control, preprint, 2018 (`arXiv:1806.01419`).

[19] E. Panteley and A. Loria, On global uniform asymptotic stability of nonlinear time-varying systems in cascade, *Systems and Control Letters*, vol. 33, no. 2, pp. 131-138, 1998.

[20] Parker Automation, *Compumotors Virtual Classroom, Position Systems and Controls*, Training and Product Catalog, CD-ROM, 1998.

[21] V. Petrović, R. Ortega, and A. M. Stanković, Interconnection and damping assignment approach to control of PM synchronous motors, *IEEE Transactions on Control Systems Technology*, vol. 9, no. 6, pp. 811–820, 2001.

[22] P. Seibert and R. Suarez, Global stabilization of nonlinear cascaded systems, *Systems and Control Letters*, vol. 14, pp. 347-352, 1990.

[23] P. Tomei and C. M. Verrelli, A nonlinear adaptive speed tracking control for sensorless permanent magnet step motors with unknown load torque, *Int. J. Adapt. Control Signal Process.* vol. 22, pp. 266-288, 2008.

[24] P. Tomei and C. M. Verrelli, Observer-based speed tracking control for sensorless PMSMs with unknown load torque, *IEEE Trans. Automatic Control*, vol. 56, no. 6, 2011.

[25] A. van der Schaft, $\mathcal{L}_2$-*Gain and Passivity Techniques in Nonlinear Control*, 3rd ed, Springer, 2017.

[26] A. van der Schaft and T. Stegink, Perspectives in modeling for control of power networks, *Annual Reviews in Control*, vol. 41, pp. 119-132, 2016.

[27] M. Zhang, L. Borja, R. Ortega, Z. Liu and H. Su, PID passivity–based control of port-Hamiltonian systems, *IEEE Trans. Automatic Control*, vol. 63, no. 4, pp. 1032-1044, 2018.

[28] D. Zonetti, R. Ortega and A. Benchaib, Modeling and control of HVDC transmission systems: From theory to practice and back, *Control Engg. Practice*, vol. 45, pp. 133-146, 2015.